Understand the Real World of Mixing

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POST MIXING OPTIMIZATION AND SOLUTIONS

Most chemical engineering curricula do not adequately address mixing as it is commonly practiced in the chemical process industries. This article attempts to fill in some of the gaps by explaining flow patterns, mixing techniques, and the turbulent, transitional and laminar mixing regimes.

For most engineers, college memories about mixing are limited to ideal reactors. Ideal reactors are extreme cases, and are represented by the perfect-mixing model, the steady-state perfect-mixing model, or the steady-state plug-flow model (Figure 1). These models help determine kinetics and reaction rates.

For an ideal batch reactor, the perfect-mixing model assumes that the composition is uniform throughout the reactor at any instant, and gives the reaction rate as:

$$(-r_A)V = N_{A0} \times (dX_A/dt) \qquad (1)$$

The steady-state perfect-mixing model for a continuous process is similar, with the composition uniform throughout the reactor at any instant in time. However, it incorporates the inlet molar flowrate, F_{A0}:

$$(-r_A)V = F_{A0} \times X_A \qquad (2)$$

The steady-state plug-flow model of a continuous process assumes that the composition is the same within a differential volume of the reactor in the direction of flow, typically within a pipe:

$$(-r_A)V = F_{A0} \times dX_A \qquad (3)$$

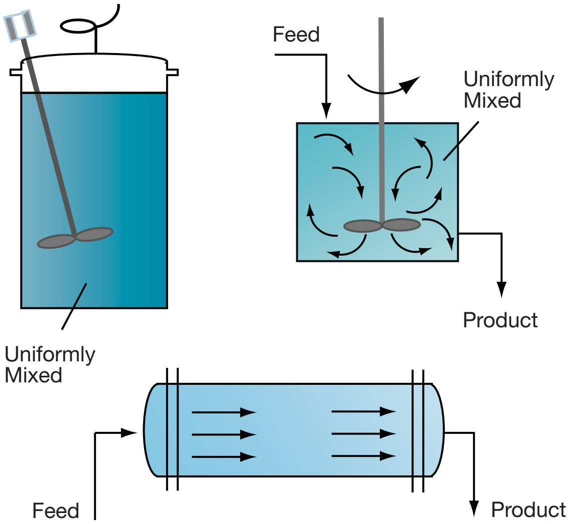
These equations can be integrated to determine the time required to achieve a conversion of X_A.

Many models are developed as combinations of the two continuous extremes to describe nonideal flow, such as in recirculation loops, short-circuiting paths, and dead zones (Figures 2 and 3) *(1)*. These extremes are seldom achieved in real life.

This is the extent of most engineers' collegiate-level preparation for real-world mixing applications. Since almost everything manufactured must be mixed, most new graduates are ill-prepared to optimize mixing processes. This article attempts to bridge the gap between the theory of the ideal and the realities of actual practice.

Limitations of the perfect-mixing and plug-flow models

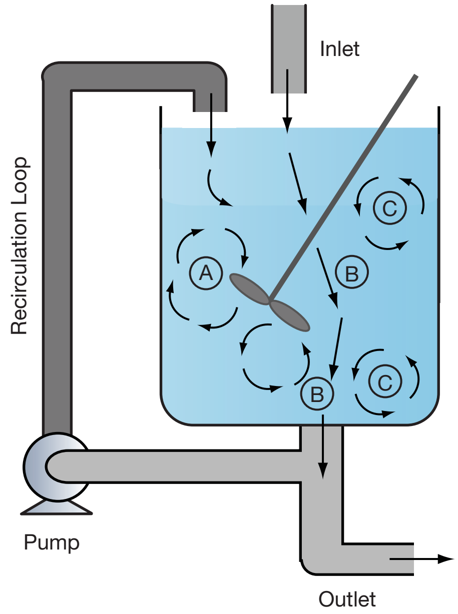
Classical chemical engineering focuses on commodity chemicals produced in large quantities in continuous operations, such as in the petrochemical, polymer, mining, fertil-



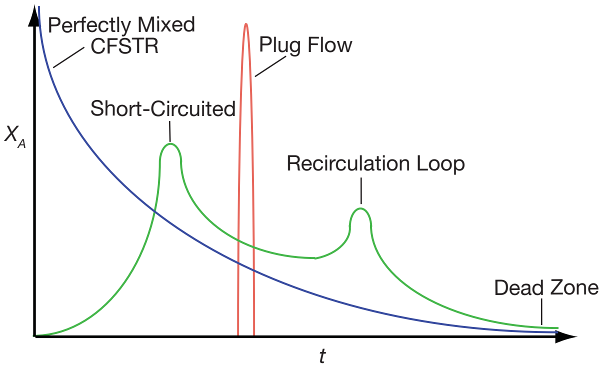
▲ **Figure 1.** The three types of ideal reactors are the perfect-mixing model (left), steady-state perfect-mixing model (right), and the steady-state plug-flow model (bottom).

izer, and pulp and paper industries. Many of these industries are mature, and their plants have already been built and optimized. In contrast, industries such as specialty chemicals, pharmaceuticals, biochemicals, personal care products, foods and beverages, and paints and coatings, as well as the emerging field of nanoscale materials, make products in much smaller volumes, employing batch processes as the main mode of mixing.

Perfect-mixing and plug-flow models imply a single-phase process. They do not address solids suspensions, liquid-liquid dispersions, gas-liquid dispersions, and high-viscosity non-Newtonian fluids, all of which are common in the chemical process industries (CPI).



▲ **Figure 2.** This schematic shows tank flow patterns, including well-mixed zones (A), the short-circuiting path (B), and dead zones (C).



▲ **Figure 3.** This compares the conversion rates of the perfectly mixed (blue) and plug-flow (red) models with real-life performance (green).

Reaction control. Engineers employ the perfect-mixing and plug-flow models to identify reaction-rate constants; this implies a kinetically controlled reaction. Although the literature contains much research on kinetics, most of it provides no proof that the process discussed was truly kinetically controlled. Consequently, most students never learn how to determine whether a process is controlled by kinetics or by mass transfer.

To prove kinetic control, it must be demonstrated that increases in impeller speed have no effect on the rate of the reaction, and such tests must be conducted at speeds well above the minimum required to carry out the reaction. Most papers in the literature state only that the reactions were run at a certain speed (which may or may not have been the minimum), without elaborating on whether the impeller speed had any impact on the reaction rate. Some papers claim that kinetic reaction rate constants vary for different impeller speeds. However, these conditions actually describe a mass-transfer-controlled reaction, not a kinetically controlled reaction.

If a process is truly kinetically controlled, it does not matter whether the impeller is a Rushton turbine, a hydrofoil, or a rotor-stator. For kinetically controlled processes, the speed of the impeller and its power input are also irrelevant. These things should be taken into account only for mass-transfer-controlled processes.

The real mixing world

The flow field of a mixing tank is three-dimensional and very complex. To get an exact account of all the velocities and shear rates in a mixing tank, computational fluid dynamics (CFD) is used to solve the partial differential equations (PDEs) associated with the Navier-Stokes momentum, energy and continuity equations *(2)*. Any solution methodology must address all three sets of PDEs simultaneously.

Analytical solutions derived directly from the PDEs are only useful for the simplest of cases. For more complicated problems, the PDEs are solved in terms of gradients using approximate solution methods, such as finite difference, finite volume, and finite element techniques. The greatest difficulty in solving these three equations is the nonlinearity of the momentum equation, which makes the resulting implementations extremely unstable. Therefore, approximations to the solutions can only be as accurate as the accuracy of the discretized domain at representing the gradients of the variables (*e.g.*, velocity, pressure, temperature, etc.).

Many CFD software programs include lines of code that introduce artificial diffusion to stabilize the nonlinear solutions. Doing this, however, may be problematic because it violates one of the Navier-Stokes equations

Nomenclature

C = constant in liquid-liquid correlation in Eq. 14
D = impeller diameter, m
$(D/T)_{opt}$ = the optimum ratio relating impeller diameter to tank diameter, dimensionless
dp = particle size, m
E = a correlation factor that collectively describes the effects of fluids other than water, as well as temperature, viscosity, and impeller design on the gas-liquid mass-transfer coefficient
F_{A0} = initial molar flowrate of substance A at time 0, mol/s
Fr = Froude number $= n^2D/g$
h_P = process-fluid-side heat-transfer coefficient, W/m^2-K
g = gravitational constant, 9.81 m/s^2
KF = factor reducing impeller power in the presence of gas = $P_{gassed}/P_{ungassed}$
k_La = gas-liquid mass-transfer coefficient, s^{-1}
n = impeller speed, rev/s
N_{A0} = initial number of moles of reactant A at time 0, mol
Nae = aeration number = $Q_G/(nD^3)$
Nae_F = aeration number at the flood point
n_{JS} = impeller speed at which the last particle is just suspended, rev/s
n_{min} = minimum impeller speed to achieve a liquid-liquid dispersion, rev/s
N_{mix} = dimensionless mixing time = $n\Theta_{mix}$
Np = power number = $P/(\rho n^3D^5)$
P_{gassed} = impeller power under gassed conditions, W
P_{imp} = impeller power, W
P_{ieg} = power of the isothermal expansion of gas bubbles, W
P_{JS} = impeller power at the just-suspended speed, W
P_{min} = minimum impeller power to achieve a liquid-liquid dispersion, W
$P_{ungassed}$ = impeller power under ungassed conditions, W
Q_G = gas flowrate, m^3/s
Q_L = liquid flowrate, m^3/s
r_A = rate of reaction based on liquid volume, mol/m^3/s
Re = Reynolds number = $(\rho nD^2)/\mu$
S = Zwietering constant in solid-liquid correlation
SV = settling velocity, m/s
t = time, s
T = tank diameter, m
V = liquid volume in the reactor, m^3
v_{sg} = superficial gas velocity, m/s
We = Weber number = $(\rho_c n^2D^3)/\mu$
x = an exponent for impeller diameter, D, that informs scale-up design
X = solids loading = 100wt.%/(100 – wt.%)
X_A = fraction of reactant A converted into product
Z = liquid level, m

Greek Letters

αLL = exponent in Eq. 14
α = factor that describes the effect of fluids other than water on gas-liquid mass transfer
ξ_{imp} = term representing the effect of the impeller design on gas-liquid mixing
ξ_{vis} = term representing the effect of the viscoscity on gas-liquid mixing
ρ = density, kg/m^3
ρ_S = solid density, kg/m^3
ρ_L = liquid density, kg/m^3
ρ_{Slurry} = slurry density, kg/m^3
ρ_c = continuous-phase density, kg/m^3
$\rho_{dispersion}$ = dispersed-phase density, kg/m^3
μ = viscosity, Pa-s
μ_c = continuous-phase viscosity, Pa-s
μ_d = dispersed-phase viscosity, Pa-s
ν = kinematic viscosity, m^2/s
σ = interfacial tension, N/m
Θ = temperature factor for gas-liquid mass transfer
Θ_{mix} = mixing time, s
Φ = gas hold-up, unitless

Subscripts

A = reactant descriptor
An = anchor
HR = helical ribbon
T = turbulent
TL = the boundary between the transitional and laminar regimes
TT = the boundary between the turbulent and transitional regimes

(usually the continuity equation).

For finite-element analysis, the Galerkin/least-squares finite element (G/L-SFE) method has a completely self-consistent and mathematically rigorous formulation that provides stability without sacrificing accuracy. Even with that approach, though, the solution is only as good as the discretized mesh. If the mesh is too coarse, the gradients will not be captured correctly; if the mesh is too fine, computational time could be unwieldy.

Correctly performed CFD analysis describes the non-ideal world of real flow situations. However, producing CFD models often consumes large amounts of time and money, and analyzing the results can be difficult for even a mixing expert. Another, simpler approach is necessary.

The rest of this article provides rules of thumb and correlations that can help you get started. The first tip: Since all of the equations discussed from this point forward include coefficients and constants based on SI units, be

Table 1. The relationship between the power number (*Np*) and the Reynolds number at turbulent/transitional boundary (Re_{TT}) is described by Equation 6.

	Np	Re_{TT}
Rushton Turbine	5.2	3,679
Bar Turbine	0.61	7,511
Pitched Bladed Turbine	1.27	5,882
Pfaudler Impeller, unbaffled	0.37	8,873
Pfaudler Impeller, 2 glass-lined baffles	0.73	7,075
A310	0.3	9,516
Propeller 1.0 Pitch	0.36	8,954
Propeller 1.5 Pitch	0.62	7,470

sure to use SI units when applying them to avoid making conversion errors.

Flow patterns

There are three basic flow patterns in mixing: tangential, axial, and radial.

For a cylindrical tank without baffles and any type of impeller on the center vertical axis, all turbulent flow will be tangential. Baffles usually consist of three or four flat plates, each of which occupies about 10% of the tank diameter, attached to the tank walls to stop tangential flow. Numerous other baffle designs that help reduce swirl exist for glass-lined reactors, but they are not as efficient as a four-flat-plate configuration.

Axial-flow impellers generate flow parallel to the axis (or shaft), either upward or downward. Radial-flow impellers create flow perpendicular to the axis or in a radial direction. Some impellers, such as the pitched-bladed turbine (PBT), create a mixed-flow pattern — *i.e.*, partly axial and partly radial.

Most available impellers satisfy the range of process requirements from flow-controlled to shear-controlled applications. Shear rate (expressed as a velocity gradient) is important, because changes in velocity are what cause mixing to occur. Tangential-, axial-, mixed-, and radial-flow impellers represent the range of shear rates from lowest to highest across the impeller spectrum. Shear rates within a reactor are best determined by CFD software analysis.

Reynolds number and power number

The Reynolds number (*Re*) describes the ratio of inertial forces to viscous forces in a fluid, indicating whether the fluid is in the turbulent, laminar, or transitional regime. For pipe flow, if $Re > 2{,}000$, the flow is considered to be turbulent.

For mixing, this classification depends on the impeller being used. The Reynolds number of an impeller is defined as:

$$Re = (\rho n D^2)/\mu \qquad (4)$$

The power number (*Np*) for all impellers, regardless of size, is defined as:

$$Np = P_{imp}/(\rho n^3 D^5) \qquad (5)$$

where P_{imp} is the amount of power that the impeller imparts to the fluid. This value is usually less than the amount of power measured at the motor and can be determined with torque cells or strain gauges on the shaft.

Transition between the turbulent and transitional regimes

The fluid is considered turbulent when its Reynolds number is greater than the Reynolds number that describes the boundary between the turbulent and transitional (*TT*) regimes *(2)*:

$$Re > Re_{TT} = 6{,}370 Np^{-1/3} \qquad (6)$$

The relationship between the power number and the Reynolds number at the boundary between the turbulent and transitional regimes is detailed in Table 1.

The power number is characteristic of the impeller design. Impellers with high *Np* (typically radial-flow impellers) begin to operate in the turbulent regime at lower Reynolds numbers. Impellers with low *Np* (typically axial-flow impellers) transition to the turbulent flow regime at much higher *Re*. Generally the transition is in the 3,000–10,000 range.

In the turbulent regime, *Np* is constant if the swirl is inhibited. In addition to using baffles, swirl can be inhibited by placing the impeller off-center, at an angle to the axis and horizon, or in square or rectangular tanks, since asymmetry helps to minimize tangential flow.

Transition between the laminar and transitional regimes

When the flow is laminar, it is best not to use common impellers. Special impellers are relatively large and extend almost to the tank wall. Baffles can hinder laminar mixing and should be avoided. In the transitional regime, the width of standard baffles is adjusted by up to one-tenth of the tank diameter.

Two correlations *(2)* predict when the flow is laminar in a mixing tank:

$$Re < Re_{TL} = 23(D/T)^{-2} Np_T^{-2/3} \qquad (7)$$

$$Re < Re_{TL} = 183 Np_T^{-1/3} \qquad (8)$$

where Np_T is defined as the turbulent power number, which is characteristic of the impeller design.

The range of Re_{TL} when calculated by Eq. 7 is $50 < Re_{TL} < 2{,}000$, whereas Eq. 8 gives a tighter range, $100 < Re_{TL} < 275$. When *Re* is less than about 50–300, the flow is likely to be laminar, but for small, low-*D*/*T* impellers ($D/T \approx 0.1$), laminar flow could extend as high as $Re = 2{,}000$. Impellers suitable for the laminar regime include anchor and helical-ribbon impellers and Ekato's Paravisc.

Mixing times

N_{mix} is a dimensionless mixing time regardless of fluid regime that is defined as the mixing time (Θ_{mix}, in units of time) multiplied by the impeller speed (*n,* units of time^{-1}). The mixing time (Θ_{mix}) may depend on a variety of factors, such as where the materials to be mixed are added, the location of the impeller, and the time required before a fluid can be considered mixed. One can generally assume the mixing time to be the time needed to reach 95% homogeneity *(2)*. It is important to understand how mixing time is defined when comparing reports from various investigators to avoid making incorrect comparisons.

N_{mix} is defined as:

$$N_{mix} = n\Theta_{mix} \qquad (9)$$

In the turbulent regime, N_{mix} is:

$$N_{mix} = 5.20T^{1.5}Z^{0.5}D^{-2}Np^{-1/3} \qquad (10)$$

When the liquid level is the same as the tank diameter ($Z/T = 1$), impellers with a larger *D*/*T* ratio have faster blend times. All impellers of the same diameter can achieve the same mixing time at the same power input *(2)*. If power is not an issue, a high-speed, high-*Np* impeller with a large *D*/*T* ratio provides the fastest mixing.

In the transitional regime *(2)*, N_{mixT} is:

$$N_{mixT} = 33{,}489(D/T)^{-2}(Re^{-1}Np^{-2/3}) \qquad (11)$$

Setting Eq. 10 equal to Eq. 11 and solving for *Re* yields Re_{TT} (Eq. 6).

If the reactor has multiple radial-flow impellers, staging may occur and the mixing time will be much longer than calculated by Eqs. 10 and 11. Staging means that each impeller's flow pattern is unaffected by the action of the other impellers. For example, three radial-flow impellers spaced more than one impeller diameter apart will behave like three separate mixing tanks.

If the reactor has multiple axial-flow impellers spaced closely enough, they will behave almost like a single impeller (*i.e.,* there will be no staging), and Eqs. 10 and 11 can be used to approximate the mixing time. If a reactor has multiple radial-flow impellers and these impellers are far enough apart to create staging, the mixing time will be longer than calculated by a factor equal to the number of staged impellers. Depending on viscosity, staging may occur if the spacing between axial-flow impellers is more than one to two impeller diameters.

Laminar mixing times are more difficult to determine and, therefore, experimentation is necessary. Mixing-time correlations for anchor and helical-ribbon impellers (denoted by the subscripts *An* and *HR*, respectively) are very specific to particular geometric aspects of the impeller and may not be applicable to other impeller geometries. Anchors have a predominantly tangential flow pattern and mix very poorly, yet they are preferred by many companies. Helical ribbons have a flat N_{mix}-*Re* curve, even in the laminar and transitional regimes, making them very efficient mixers. For example, at $Re = 200$, $N_{mix,An} = 1{,}500$ whereas $N_{mix,HR} = 50$; at $Re = 1{,}000$, $N_{mix,An} = 100$ whereas $N_{mix,HR} = 43$. Anchors are excellent for heat-transfer applications because they create tangential flow, which produces the highest flow at the walls where the heat-transfer surface is located. They do mix; they just take longer to do so.

Some rules of thumb for turbulent mixing are:

1. Use large-diameter, low-speed, axial-flow impellers.
2. Minimize the differences between viscosity and density.
3. It is easier to mix a viscous fluid into a low-viscosity liquid than it is to mix a low-viscosity liquid into a viscous fluid.
4. Use dip tubes and introduce reactants as close to the high-shear zone of the impeller as possible.
5. For processes with selectivity concerns, do not add the reactants at the liquid surface *(2)*. Instead, try to add them as close to the highest-shear zone in the reactor, which will be near the inlet or outlet flow of an impeller blade.
6. For continuous mixing, try to achieve at least three mixing times ($3\Theta_{mix}$) per residence time (V/Q_L) of the reactor to minimize short-circuiting and dead zones. For the minimum (3:1), choose impellers that create flow patterns that do not oppose the general through-flow. A ratio of 10:1 (or higher) is required to achieve the degree of mixing predicted by the perfect-mixing model.

Heat transfer

Little can be done to improve heat transfer without increasing the heat-transfer surface area. If a jacket alone is insufficient, try adding coils to improve heat transfer (inside coils alone or with the outer jacket heat-transfer surface). Mixing affects only the process-fluid-side heat-transfer film coefficient, h_p. To double h_p, power must be increased

Table 2. These heat-transfer coefficients are for organic fluids using vertical or helical tube coils.

Viscosity (cP)	Heating*	Heating with jacket*	Cooling*	Cooling with jacket*
1	3,400	2,200	1,100	700
10	1,400	900	560	360
100	560	360	300	200
1,000	250	160	170	110
5,000	150	100	110	70

* hp in units W/m^{2}K

about 23-fold! Doubling the diameter of the impeller will only increase h_P by about 10%. This relationship is fairly constant for all types of impellers at the same D/T and P/V. Since tangential-flow impellers promote the most efficient heat transfer (the highest values of h_P), the Pfaudler impeller and anchor impellers are the best choices to enhance heat transfer. The most efficient way to increase heat transfer is to install internal vertical tubes or helical coils, which provide about 54% more heat-transfer area than a jacket alone and 37% more than plate coils *(3)*.

Because heat transfer is not affected much by impeller design, the approximate values of h_P provided in Table 2 can be used regardless of the type of impeller. These values are for organic fluids in vertical tubes or helical coils and reactor jackets. To obtain heat-transfer coefficients for aqueous fluids, multiply the values in Table 2 by a factor of 3–4, since aqueous fluids have higher heat capacities.

Solid-liquid mixing

For reactions to occur efficiently, solids must rise from the tank bottom into suspension. To achieve solids suspension, axial- and mixed-flow impellers placed at about one-quarter of the liquid level (from the bottom) are commonly used.

Figure 4 can be used to determine an application's settling regime. Numerous correlations in the literature describe the free-settling solids regime *(3)*, but the Zwietering correlation *(2)* is most often referenced. Solids in the hindered-settling regime behave as though they were in a viscous fluid application, and generalized correlations do not exist. Cases with hindered settling require experimentation and careful scale-up.

The Zwietering correlation is used to determine the minimum speed required to suspend all solids off the bottom of the tank, n_{JS}:

$$n_{JS} = S\rho^{0.1}[g(\rho_s - \rho_L)/\rho_L]^{0.45} X^{0.13} d_P{}^{0.2} D^{-0.85} \qquad (12)$$

The minimum just-suspended speed determined by Eq. 12 is then used to calculate the minimum power required to suspend all solids off the bottom of the tank, P_{JS}:

$$P_{JS} = \rho_{Slurry} Np\, n_{JS}{}^3 D^5 \qquad (13)$$

In Eqs. 12 and 13, X is the solids loading. The constant S is a function of impeller design and location above the tank bottom and it can be estimated as $S = 4$ for PBT impellers and $S = 7$ for hydrofoils. More values of S can be found Ref. 2.

It is not possible to achieve truly uniform suspensions — the top surface will always be void of the heaviest particles. Uniformity is considered to have been achieved when further increases in mixer power no longer affect the degree of suspension. Uniform suspensions are rarely required, unless the process is continuous and there is an overflow configuration. The additional power required to achieve uniformity depends on the solids' settling velocity (SV).

For settling velocities less than 0.003 m/s, uniformity occurs when the last particle is lifted off the bottom of the vessel. This is considered an easy suspension application. Within the range $0.02 < SV < 0.04$ m/s, three times more power is required to achieve uniformity. This is considered a moderately difficult application. When $SV > 0.08$ ms, the application is considered difficult, requiring five times the power for suspension.

Increasing power beyond P_{JS} does not significantly improve mass transfer. Doubling the power will increase the mass-transfer coefficient by only about 19%. In a difficult application, increasing the power enough to achieve a uniform (rather than just off-the-bottom) suspension would increase the mass-transfer coefficient by only 50%.

Axial-flow impellers provide flow in the axial direction, parallel to the impeller shaft and perpendicular to the blades. An axial-flow impeller can provide flow in either a downward or an upward axial direction.

Some rules of thumb for dealing with axial-flow impellers:

- for down-pumpers (which create downward flow), $0.25 < (D/T)_{opt} < 0.45$
- for up-pumpers (which create upward flow), $0.45 < (D/T)_{opt} < 0.7$
- operate at a speed that is a few rpm above the predicted n_{JS}.

Liquid-liquid mixing

In the absence of agitation, two immiscible liquid phases will be separated. The minimum speed required to create a dispersion with no puddles of aqueous or organic fluids is given by *(2)*:

$$n_{\min} = C\left(\frac{T}{D}\right)^{\alpha LL}\left(\frac{\mu_c}{\mu_d}\right)^{1/9}\left(D^{-0.5}\right)\left(\frac{g^2\Delta\rho}{\rho_c}\right)^{0.25}\left(\frac{\sigma}{D^2\rho_c g}\right)^{0.3} \qquad (14)$$

$$P_{\min} = \rho_{dispersion}(Np)n_{\min}^3 D^5 \qquad (15)$$

Values for C and α for an axial-flow, a mixed-flow, and two radial-flow impellers at various off-bottom clearances have been published *(2)*. For a Rushton turbine located at the interface between the two settled fluids, $C = 4.0$ and $\alpha LL = 0.88$. Radial-flow impellers such as the Rushton turbine are commonly used for liquid-liquid mixing because they provide the high shear required to form droplet dispersions.

Unlike when dealing with solids suspension, increasing power beyond P_{min} can be beneficial for the mass transfer between two liquid phases, because more power, and thus more shear, reduces the droplet diameter. Doubling the power will result in a 17–59% increase in the liquid-liquid mass-transfer coefficient.

It is important that mixing not be too vigorous, because this could produce a stable emulsion. Entrainment of one phase into the other can reduce the product purity, which happens when excessive power and shear create extremely fine droplets (fines). The presence of solids or gas can stabilize an emulsion, whereas an increase in temperature can help break one up.

Some industries, such as cosmetics and biochemicals, need to make stable emulsions. In those applications, an enormous amount of shear is desired; mixing is often done with rotor/stators (also known as high-shear mixers). In a rotor/stator, the rotating impeller is very close to a stationary element, which is usually attached to a stationary support structure that surrounds the rotating shaft. The impeller (rotor) and the stationary element (stator) often have grooves and ridges or slots through which the fluid is forced to increase the shear. Rotor/stators are the highest-shear devices available. The mixing theory on rotor/stators has not yet matured, and the best advice remains trial and error.

Gas-liquid mixing

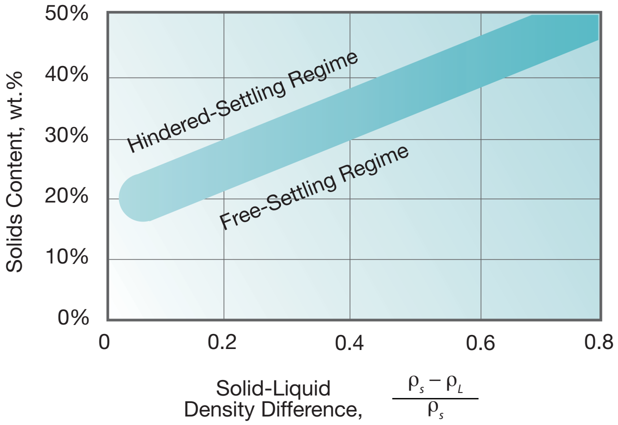
In order for gas-liquid mixing to be effective, the gas bubbles need to be dispersed by the impeller. When this fails to occur, the impeller is referred to as being flooded. This condition can be identified by wild geysering and burping of gas through the liquid surface. Gas-liquid mixing can be evaluated by three different methods.

The rule-of-thumb method considers the impeller power and the power of the isothermal expansion of the gas bubbles (P_{ieg}) by comparing P_{imp}/V with P_{ieg}/V. P_{ieg}/V is a function of the superficial gas velocity (v_{sg}); when using SI units, a good approximation is $P_{ieg}/V = 9{,}843v_{sg}$. Then:

- the impeller is flooded when $P_{imp}/V < P_{ieg}/V$
- the gas is uniformly dispersed when $P_{imp}/V > 3\ P_{ieg}/V$
- when P_{imp}/V is two to three times larger than P_{ieg}/V, the gas is dispersed above but not below the impeller.

Another method, which is applicable to Rushton turbines, is based on the aeration (*Nae*) and Froude (*Fr*) numbers *(2)*. Here:

- the impeller is considered flooded when $Nae > 30Fr(D/T)^{3.5}$ or when $N < 0.033Q_G^{1/3}g^{1/3}D^{-4/3}(D/T)^{-1.17}$



▲ **Figure 4.** Comparing the solid weight percentage with the difference between the solid and liquid densities describes both the hindered-settling and free-settling regimes.

- the gas is uniformly dispersed when $Nae < 13Fr^2(D/T)^5$ or when $N > 0.077Q_G^{0.2}g^{0.4}D(D/T)^{-5}$.

When an axial-flow impeller's aeration number at the flood point, Nae_F, is constant, it may be characterized by this value. When *Nae* is greater than Nae_F, the impeller is assumed to be flooded, whereas when *Nae* is less than Nae_F, it is assumed to be dispersed. Typical values of Nae_F are 0.055 for high-solidity hydrofoils, 0.02–0.035 for PBT, and 0.01–0.02 for low-solidity hydrofoils.

The presence of gas around the impeller usually reduces the impeller's power draw (compared with the same impeller in the absence of gas). The degree of reduction is referred to as the *K* factor, where $KF = P_{gassed}/P_{ungassed}$. Numerous correlations are available to determine the *K* factor, but there are too many to present here; Ref. 2 provides more information. Depending on the gas flowrate (Q_G), the *K* factor of Rushton turbines and PBTs can be as low as 0.3 or as high as about 0.6. The presence of gas has little effect on newer radial-flow impeller designs that have profiled blades (such as the Smith turbine, BT-6, and PhaseJet) and up-pumpers, which often have *K* factors in the range of 0.8–0.95. Hydrofoils often have *K* factors above 1 at lower *Nae* and *K* factors about 0.75 just before they flood.

The liquid surface of gassed reactors rises because of the dispersed gas volume that is retained in the liquid, which is known as the gas hold-up, Φ. This increased volume needs to be considered during design so the reactor does not overflow or is not filled beyond the exhaust pipes and condensers. An approximation for the gas hold-up (for all mixers, including those with multiple impellers on the same shaft) is *(2)*:

$$\Phi = 0.9(P_g/V)^{0.2}v_{sg}^{\ 0.55} \qquad (16)$$

Gas-liquid mass transfer is most efficient when the impeller is not flooded and the gas bubbles are dispersed throughout the reactor. The mass-transfer coefficient, k_La, is usually described by a similar equation:

$$k_La = 1.2E(P_g/\rho V)^{0.7}v_{sg}^{0.6} \qquad (17)$$

where E is a correlation factor that collectively describes the effect of fluids other than water, as well as the effect of temperature, viscosity, and impeller design on the gas-liquid mass-transfer coefficient, k_La:

$$E = \alpha\Theta^{(t-20)}\xi_{vis}\xi_{imp} \qquad (18)$$

where the terms ξ_{vis} and ξ_{imp} represent the effect of fluid viscosity and impeller design, respectively.

If the fluid behaves like water, the temperature is 20°C, the liquid's viscosity is less than 10 mPa, and the impeller design is fairly standard, then $E = 1$. The α term describes the change in k_La due to the fluid being different (assuming otherwise identical conditions). A good approximation for the temperature factor is $\Theta = 1.024$. At temperatures above 20°C, k_La will be higher. The effect of viscosity, ξ_{vis}, is proportional to $\mu^{-0.7}$ for $\mu > 40$ mPa. The effect of the impeller design, ξ_{imp}, is relevant if it can be clearly demonstrated that one impeller provides better mass transfer than most other impellers; otherwise, $\xi_{imp} = 1$. If multiple impellers are mounted on the same shaft, $\xi_{imp} < 1$.

Doubling the power in a gassed situation increases k_La by 62% in the nonflooded gassing regime. Increasing power makes sense until the gas that is being transferred is consumed.

Scale-up and scale-down

Scaling up is not always easy because the conditions that are optimum at the small scale are often not optimum at a larger scale. Instead of developing the process in the laboratory and trying to scale it up, it is better to think about scaling down based on large-scale reactors at your disposal. This will define the limits of the impeller speed, power, and, in some cases, the impeller design. If you can determine what factors are responsible for the success of a process, the following guidelines, which are based on geometrical similarity and the relationship nD^x = constant *(2)*, can be used for scaling up the process:

- if $x = 2.0$, then constant Reynolds number and constant heat transfer are important
- if $x = 1.5$, then constant Weber number (which is a measure of the relative importance of the fluid's inertia compared to its surface tension, $We = (\rho_c n^2 D^3)/\mu$) and dispersed-phase drop size are important
- if $x = 1.0$, then constant tip speed and maximum shear rate are important
- if $x = 0.85$, then off-bottom suspension is important
- if $x = 0.67$, then mass transfer, drop dispersions, reactions, and gas hold-up, as well as holding P/V constant, are important
- if $x = 0.5$, then constant Froude number, vortex formation, gas dispersion state, and suspension of droplets are important
- if $x = 0.0$, then constant impeller speed, equal mixing time, and average shear rate are important.

It is important to not work at a scale that is too small for three reasons. First, since Re is proportional to nD^2, the fluid regime may not be turbulent at small scale, but it probably will be at a large scale. It is a good idea to maintain the fluid regime during scale-up. Second, the dispersed phase in multiphase mixing applications may be relatively large compared to the impeller dimensions, so the dispersed phase will not experience the same shear rate spectrum at small scale that it would at a large scale. Finally, the small-scale configuration mixes so well that maintaining geometrical similarity is insufficient for proper scale-up. Upon scale-up, the location of the feed points have a greater impact on mixing, compared with small-scale endeavors.

In closing

The perfect-mixing and plug-flow models are insufficient for describing mixing in real-world applications. The rules of thumb presented here are no substitute for experience in mixing, but they should give new engineers a head start and provide seasoned professionals with a good review of important concepts.

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